

Overturning Circulations

(or: why the Hadley and Ferrel cells exist)

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1 Introduction

One of the most prominent features of global climate is the large-scale patterns of atmospheric circulation. These global circulations play a crucial role in redistributing water and energy across the earth. In this set of notes we focus on the overturning circulations: the Hadley and Ferrel cells. We'll explore why large-scale circulations are needed to redistribute energy (as opposed to a planet in radiative convective equilibrium alone), outline a few theories of how these circulations are maintained (eddy transport vs. conservation of angular momentum), and see what we can learn from simple models of these circulations.

Because we are interested in circulations that transport water and energy meridionally, we will use zonally averaged quantities in the following sections. Although the Hadley cell does have zonal asymmetries, we will neglect them for the time being. This will leave us with a more transparent model that is still useful for understanding why overturning circulations exist, and how they are maintained.

As a point of reference, these notes follow closely from Vallis (2006), and so for a more detailed treatment of the topics discussed below, I refer readers to Chapter 11 of Vallis (2006).

2 Large scale features of the atmosphere

Here we present some basic observations that establish why overturning circulations exist, and what they look like in our present-day climate. These observations will motivate our further investigation into the dynamics of these circulations using a few simple models.

2.1 Radiative equilibrium

The most basic measure of how overturning circulations affect the global climate is the equator-to-pole distribution of energy. One way to observe this in the current climate is to compare estimates of incoming solar radiation to those of outgoing infrared radiation. In other words, where is energy coming into the system, and where is it leaving the system. If the earth were in perfect radiative convective equilibrium (i.e. no atmosphere or ocean circulations), we would expect these two curves to coincide perfectly. However, as panel (a) in the figure below illustrates, in today's climate that's not the case. The earth takes in more radiation than it emits in the tropics and subtropics, and emits more radiation than it receives at high latitudes. This simple diagram demonstrates how important overturning circulations are in the present climate.

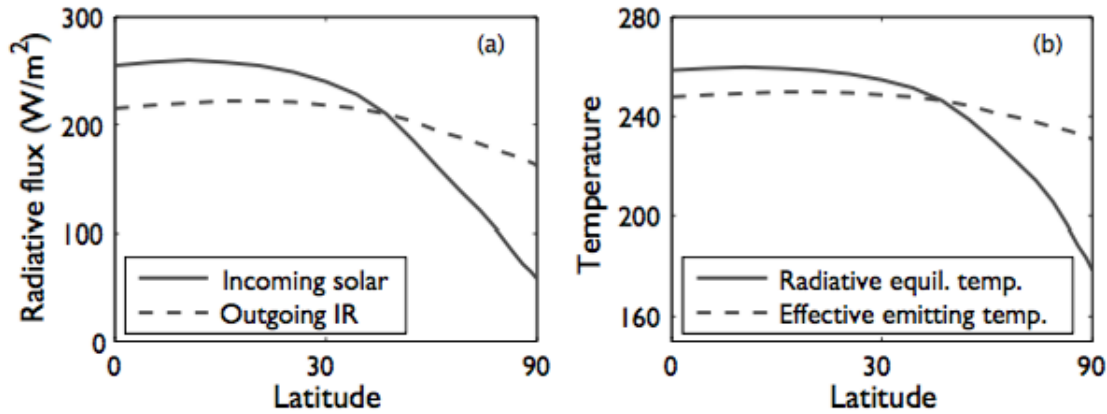


Fig. 11.1 (a) The (approximate) observed net average incoming solar radiation and outgoing infrared radiation at the top of the atmosphere, as a function of latitude (plotted on a sine scale). (b) The temperatures associated with these fluxes, calculated using $T = (R/\sigma)^{1/4}$, where R is the solar flux for the radiative equilibrium temperature and R is the infrared flux for the effective emitting temperature. Thus, the solid line is an approximate radiative equilibrium temperature

Figure 1: Credit: Vallis (2006)

We can further convert the energy distribution depicted in panel (a) above into temperature for the hypothetical case of radiative convective equilibrium, and compare it to observations (see panel (b) in the figure above. This recasts the problem in terms of the equator-to-pole temperature gradient.

2.2 Zonal winds

We can predict a great deal about the zonal flow using only the thermal wind balance and the observed equator-to-pole temperature gradients that we discussed in the previous section. Remember that the thermal wind balance is

$$f \frac{\partial u}{\partial p} = \frac{R}{p} \frac{\partial T}{\partial y}$$

which relates the vertical wind shear to meridional gradients in temperature. To get a full picture of the zonal wind, however, we should make a few additional notes about the structure of meridional temperature gradients throughout the atmosphere. In the troposphere, temperature falls monotonically with latitude. This gradient is larger in the winter hemisphere than the summer because peak insolation is shifted towards the summer hemisphere, so the winter pole receives virtually no direct sunlight. Above the troposphere (starting somewhere between 8 and 16 km above the sea level) is the stratosphere, where temperature

increases with height. In the stratosphere the meridional temperature gradient is reversed (or, for the upper stratosphere, monotonic from pole-to-pole).

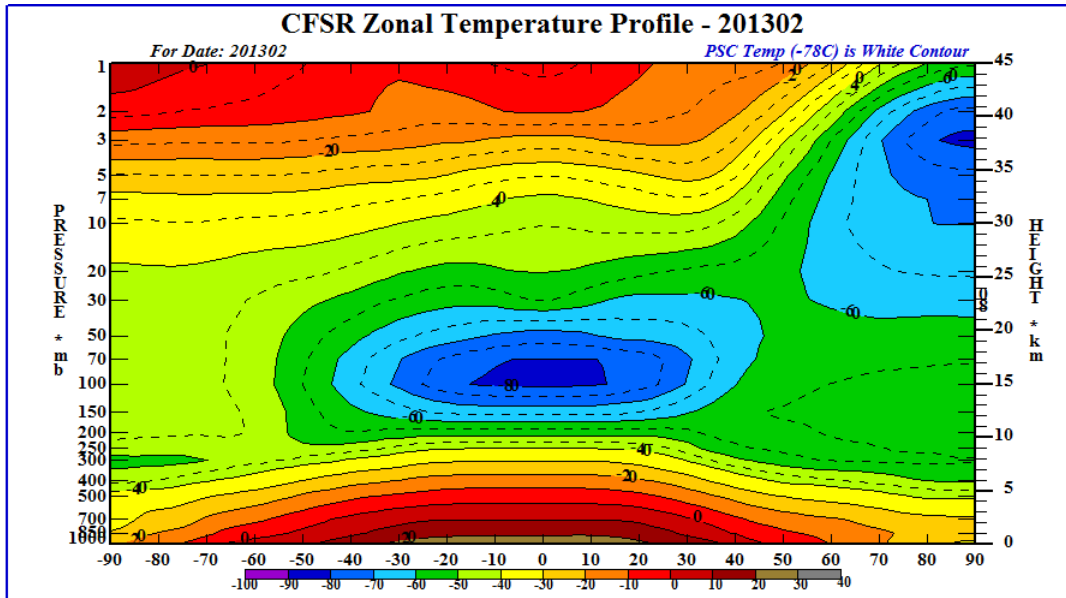


Figure 2: Stratospheric and tropospheric zonal temperature profile during northern hemisphere winter (taken in February).

Credit: National Weather Service, Climate Prediction Center website www.cpc.ncep.noaa.gov/products/stratosphere/theta/theta_info.shtml

By mentally applying the thermal wind relation to the above figure and comparing it to observations of zonal wind in the below figure, we can see the qualitative resemblance (note that the zonal wind figure reaches only 30 km while the temperature figure reaches 45 km). In the troposphere, meridional temperature gradients are largest at the edge of the subtropics, which leads to zonal jets, with the winter hemisphere jet being stronger than the summer hemisphere jet. The zonal winds also follow expectations in the stratosphere.

At the surface, winds alternate from easterly near the equator to westerly in the midlatitudes and finally easterly again at high latitudes. Surface winds are stronger in the southern hemisphere due to a lack of drag from continental land masses. And while only one season is shown here, surface winds within a given hemisphere are stronger in the winter as compared to the summer season (corresponding to the stronger jet aloft).

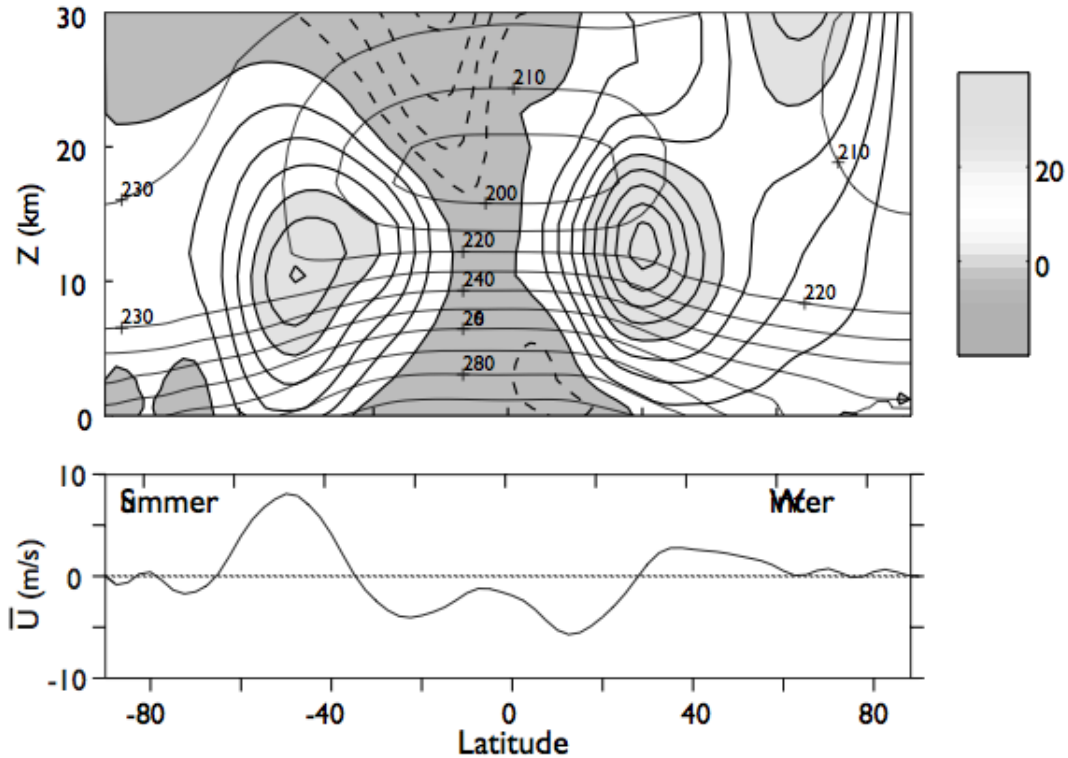


Figure 3: Top panel: zonally averaged zonal wind, bottom panel: zonally averaged zonal winds at the surface
 Credit: Vallis (2006)

A significant result of these observations is that there is no need to invoke a convergence of momentum to drive these jets. They may result from the temperature gradient and thermal wind alone. However, just as the observed meridional temperature gradient is smaller than predicted by radiative convective equilibrium, by extension the zonal wind shear is also smaller than what would be predicted by radiative convective equilibrium. The net impact of large scale circulations, particularly turbulent motions in the mid-latitudes, is to reduce the amplitude of temperature and zonal wind shear via poleward energy transport.

2.3 Overturning circulations

We won't derive the structure of the Hadley cell just yet, but will instead make few very brief observations about overturning circulations in general. We often conceptualize overturning circulations using seasonal (or annual) averages of atmospheric flow. And as with any unsteady flow, this picture may be misleading,

particularly in regions where eddy motion is important (In our case, eddy motion is important in the mid- to high-latitudes). With this caveat in mind, we can make a few notes about the seasonally averaged streamlines depicting the overturning circulation below

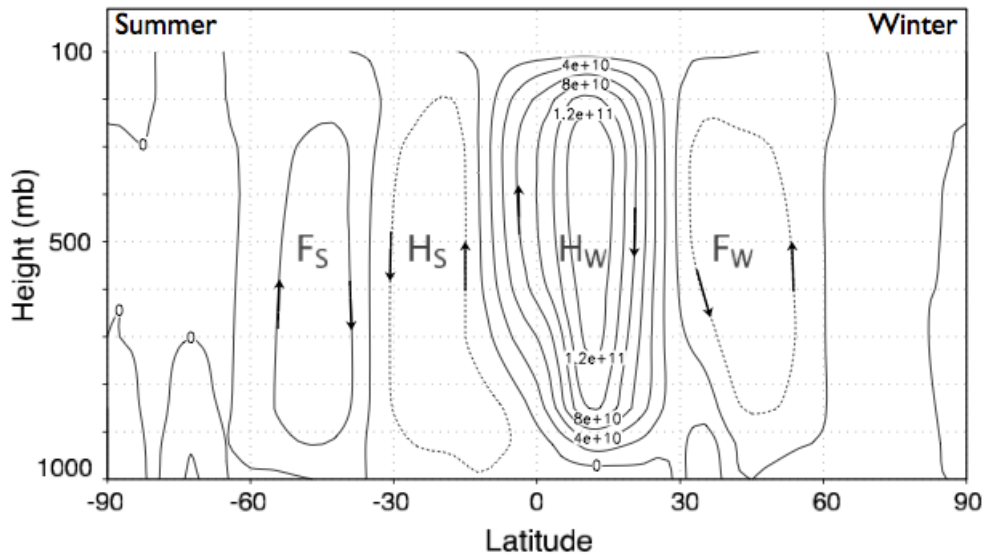


Fig. 11.3 The observed, zonally averaged, meridional overturning circulation of the atmosphere, in units of kg s^{-1} , averaged over December–January–February (DJF). In each hemisphere note the presence of a direct *Hadley Cell* (H_W and H_S in winter and summer) with rising motion near the equator, descending motion in the subtropics, and an indirect *Ferrel Cell* (F_W and F_S) at mid-latitudes. There are also hints of a weak direct cell at high latitudes. The winter Hadley Cell is far stronger than the summer one.

Figure 4: Credit: Vallis (2006)

The meridional overturning circulation of the earth has two primary components: (1) the Hadley cell extending from the tropics through the subtropics and (2) the Ferrel cell, extending through the mid-latitudes. The Hadley cell is a ‘thermally direct’ circulation, meaning that rising motion is associated with relatively warmer parcels, and sinking motion with relatively cold parcels. The Hadley cell is not hemispherically symmetric, instead the winter-cell is far stronger than the summer cell. The Ferrel cell is a thermally indirect circulation: the relatively warm parcels sink while the cooler parcels at higher latitudes rise, but remember that eddy motion is likely important in much of this region.

3 A simple model of the Hadley cell

In this section we will derive a model of the poleward extent of the zonally symmetric Hadley cell. The basic model will be one of air rising at the equator, moving polewards aloft to some latitude, ϑ_H , and returning to the equator as surface flow. We will assume that (1) the flow is steady, (2) the meridional flow aloft conserves angular momentum while the zonal flow at the surface is dissipated by friction and so is weak and finally (3) the circulation is in thermal wind balance. We therefore begin with the zonally averaged Boussinesq momentum equation in polar coordinates:

$$\frac{\partial \bar{u}}{\partial t} - (f + \bar{\zeta})\bar{v} + \bar{w} \frac{\partial \bar{u}}{\partial z} = -\frac{1}{a \cos^2 \vartheta} \frac{\partial}{\partial \vartheta} (\cos^2 \vartheta \overline{u'w'}) - \frac{\partial \overline{u'w'}}{\partial z}$$

where ζ is the relative vorticity and ϑ refers to latitude (not potential temperature!). If we neglect the vertical advection term on the left hand side, as well as the eddy terms on the right hand side and search for a steady solution, we must satisfy

$$(f + \bar{\zeta})\bar{v} = 0$$

Remember that $f = 2\Omega \sin \vartheta$ and, in polar coordinates, $\bar{\zeta} = -\frac{1}{(a \cos \vartheta)} \frac{\partial}{\partial \vartheta} (\bar{u} \cos \vartheta)$. Now if we assume that the meridional velocity is not zero, and if we apply the chain rule to $\bar{\zeta}$, then we can rewrite $f = -\bar{\zeta}$ as

$$2\Omega \sin \vartheta = \frac{1}{a} \frac{\partial \bar{u}}{\partial \vartheta} - \frac{\bar{u} \tan \vartheta}{a}$$

Now at the equator we'll assume that $\bar{u} = 0$ because parcels must rise from the surface, where we have assumed the flow is weak. So if we apply this assumption, we can solve the above equation to find

$$\boxed{\bar{u} = \Omega a \frac{\sin^2 \vartheta}{\cos \vartheta}} \quad (1)$$

This is an equation for the zonal wind in the upper branch of the Hadley circulation (remember that the zonal wind in the lower branch will be close to zero). We can infer from this equation that the zonal wind will increase with latitude. While this equation is interesting, it does not provide any constraints on the flow, only a description of the flow. To develop constraints on where this solution is valid, we will need to transform this equation into a conserved variable (potential temperature) using the thermal wind balance. By subbing the above equation into the thermal wind equation (note that we've skipped a few steps of algebra), we get:

$$\frac{1}{a \theta_a} \frac{\partial \theta}{\partial \vartheta} = \frac{2\Omega^2 a \sin^3 \vartheta}{gH \cos \vartheta}$$

Remember again that ϑ refers to latitude, while θ refers to potential temperature. Now if we integrate from the surface to the top of the atmospheric

column, H , and make use of a small-angle approximation (thus assuming the latitudinal extent of the Hadley Cell is not too great), we are left with:

$$\theta = \theta(0) - \frac{\theta_0 \Omega^2 y^4}{2gHa^2} \quad (2)$$

This equation tells us that the potential temperature in an atmospheric column begins at θ_0 on the equator, and declines poleward as the zonal wind increases. For the remainder of this derivation, rather than move through the algebra (see Vallis (2006) for this), we will outline our arguments in words and with a figure.

We have now derived an equation for the poleward transport of potential temperature by a meridional overturning circulation, and we will now try to find how far poleward the Hadley circulation extends. Finding the latitudinal extent of the Hadley Cell is equivalent to finding the domain in which equation 2 is valid. So let's begin by remembering that earlier we referred to the theoretical profile of potential temperature for an atmosphere in radiative convective equilibrium (Fig. 1). Because potential temperature is conserved, the integral of these two curves must be equal (i.e. the Hadley cell can advect energy poleward, but it cannot not create or destroy it). So to solve for the terminating latitude of the Hadley cell we will find the latitude at which the integral of the two potential temperature profiles – the radiative convective equilibrium profile and the angular momentum-conserving profile – are equal to one another. We depict this argument visually in the figure below

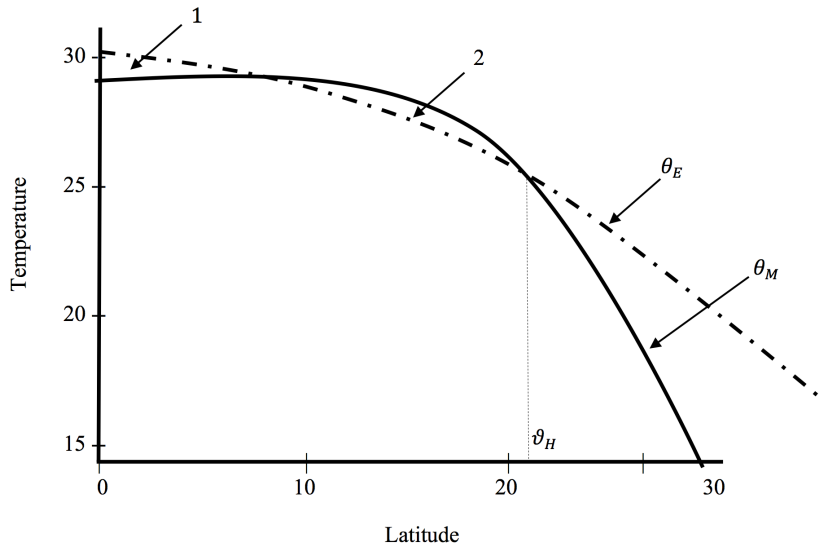


Figure 5: Meridional extent of the Hadley cell as inferred using two profiles of potential temperature - an angular momentum-conserving profile θ_M and a radiative convective equilibrium profile θ_E . Our derived solution is valid for a Hadley cell that terminates at the latitude where the integrals of the two curves are equal to one another (i.e. area labeled "1" is equal to area "2"). This latitude is ϑ_H .

For a mathematical derivation of the above arguments see Vallis, Chapter 11. This model is important for any number of reasons, but primarily because (1) it gives us a meridional structure to a potential temperature profile, which tells us that the Hadley cell tends to decrease potential temperatures in the tropics, and increase them in the mid latitudes and (2) it tells us that we need not invoke baroclinic instability in the mid-latitudes as a mechanism for terminating the Hadley cell in the subtropics. It will terminate due to conservation of angular momentum alone. This doesn't mean that in the real world eddy motion is not important for terminating the Hadley cell (it likely is), only that even without it the Hadley cell would be finite in its meridional extent.

3.1 Eddies and the Hadley Cell

An alternative conceptual model of the Hadley cell is one driven by eddies. The logic here is that as the thermal wind balance drives a vertical wind shear, the strong wind shear of the atmosphere becomes unsustainable, leads to unsteady flow and eventually to baroclinic instability. When this happens the baroclinic motions transport heat and momentum poleward, and so angular momentum is no longer conserved (as we assumed in the previous model). Let's think a little

more about how a divergence of heat and momentum in the subtropics would affect the overturning circulation.

We know that the angular-momentum conserving zonal wind would be weakened if angular momentum were not conserved as we move poleward. So in this sense eddies lead to a weaker overturning circulation. At the same time, however, the vertical component of eddy flux divergence will act to strengthen the circulation. Because the circulation is ultimately driven by equator-to-pole temperature gradients, we can also explore the impacts of eddy heat transport on the overturning circulation. Eddies lead to a divergence of heat in the subtropics, which will increase the local meridional temperature gradient. Increasing the meridional radiative convective equilibrium temperature gradient will act to strengthen the overturning circulation. So we have a number of competing effects, and the magnitude of each term is not immediately clear.

To solve this conundrum, we can make use of global climate models, which demonstrate that the strength of the overturning circulation increases proportionally with the strength of eddies in the model.

4 The Ferrel cell

In what follows I'll outline the major conceptual points of why we have a thermally indirect Ferrel cell. I'll skip the derivations but they can be found in Holton and Hakim (2012), Chapter 10. Our starting point will be the zonal-mean zonal momentum and thermodynamic energy equations for quasi-geostrophic motion on a β plane:

$$\begin{aligned}\frac{\partial \bar{U}}{\partial t} - f_0 \bar{v} &= -\frac{\partial(\overline{u'v'})}{\partial y} + (\text{zonal drag}) \\ \frac{\partial \bar{T}}{\partial t} - N^2 H R^{-1} \bar{w} &= -\frac{\partial(\overline{v'T'})}{\partial y} + (\text{adiabatic heating})\end{aligned}$$

From these equations alone, we can see that in steady state ($\frac{\partial}{\partial t} = 0$) our dominant balances are going to be:

- Coriolis force \approx divergence of eddy momentum fluxes
- Adiabatic cooling \approx Eddy heat flux convergence + diabatic heating

Holton goes on to define a mean meridional stream function ($\bar{\chi}$) using the above equations:

$$\bar{\chi} \propto -\frac{\partial}{\partial y}(\text{adiabatic heating}) + \frac{\partial^2}{\partial y^2}(\text{eddy heat flux}) + \frac{\partial^2}{\partial y \partial z}(\text{eddy momentum flux}) + \frac{\partial}{\partial z}(\text{zonal drag}) \quad (3)$$

such that the MCC stream function is positive for the Hadley cell and negative for the Ferrel cell (see Figure 6).

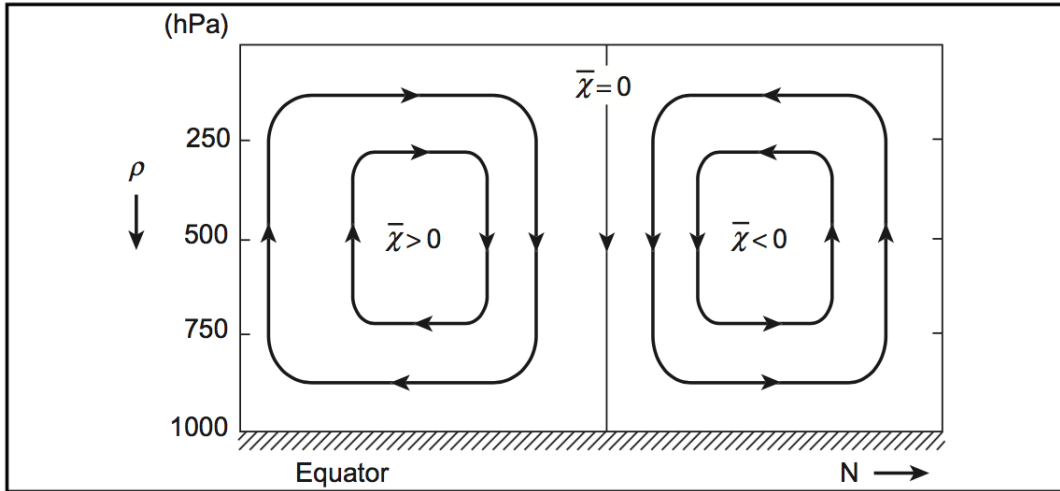


FIGURE 10.1 Relationship of the Eulerian mean meridional streamfunction to vertical and meridional motion.

Figure 6: The mean meridional circulation stream function derived by Holton and Hakim (2012).

Now, from here we can look at how the eddy heat and momentum fluxes, which arise due to instabilities in the mid-latitudes, affect the mean meridional circulation in the midlatitudes poleward of the Hadley cell ($\sim 30\text{-}75^\circ \text{ N/S}$). To do this we need to consider the observed large-scale eddy fluxes of heat (Figure 7) and momentum (Figure 9). As we will demonstrate, both the eddy heat flux and the eddy momentum flux independently act to drive the Ferrel cell. We therefore often refer to the Ferrel cell as being driven by baroclinic instability, in contrast to the thermally direct Hadley cell.

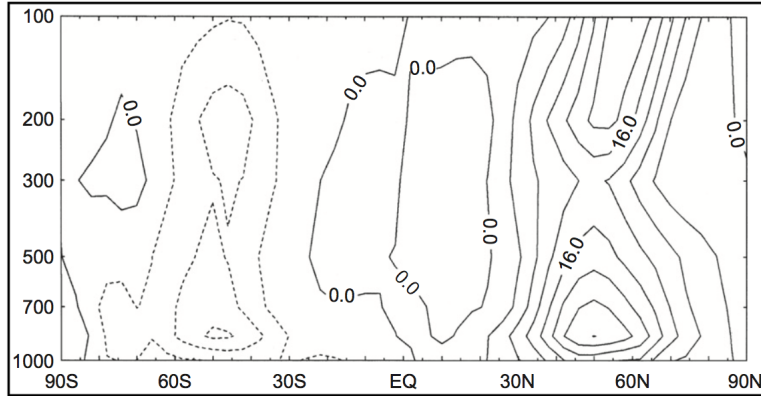


FIGURE 10.3 Observed northward eddy heat flux distribution ($^{\circ}\text{C m s}^{-1}$) for Northern Hemisphere winter. (Adapted from Schubert *et al.*, 1990.)

Figure 7: Holton and Hakim (2012).

As indicated in our equations above, the eddy heat flux convergence / divergence is going to be balanced by adiabatic heating / cooling. So from Figure 7 we can see that in the northern hemisphere there is convergence of $v'T'$ poleward of 50N and divergence equatorward of 50N. We can therefore infer adiabatic cooling (ascent) toward the pole and descent toward the equator. Continuity then dictates that we have equatorward flow aloft and poleward flow at the surface. This is depicted in Figure 8

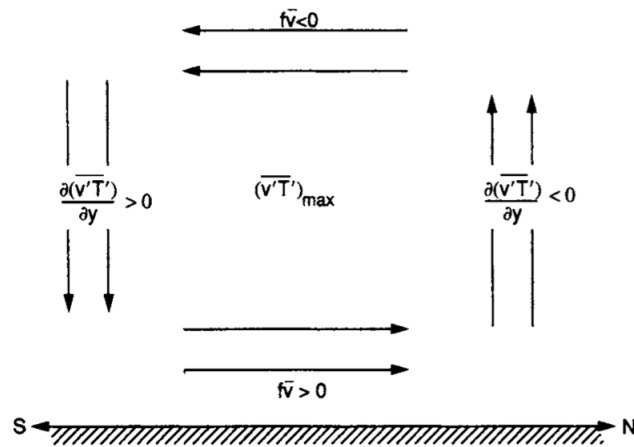


Fig. 10.4 Schematic Eulerian mean meridional circulation forced by poleward heat fluxes.

Figure 8: Holton and Hakim (2012).

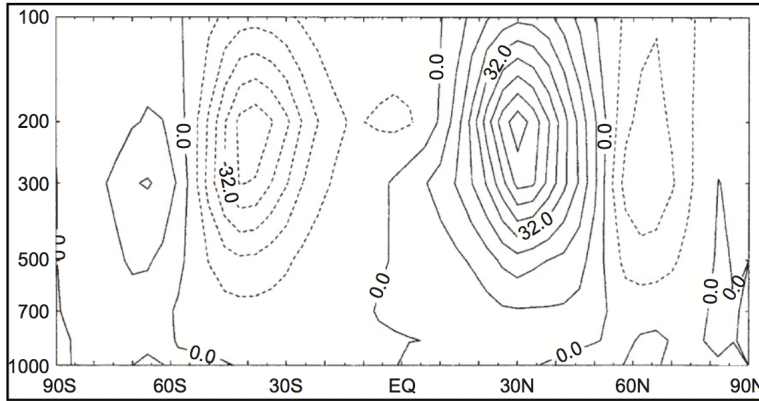


FIGURE 10.6 Observed northward eddy momentum flux distribution ($\text{m}^2 \text{s}^{-2}$) for Northern Hemisphere winter. (Adapted from Schubert et al., 1990.)

Figure 9: Holton and Hakim (2012).

Next we can consider how eddy momentum fluxes are altering the large-scale mean meridional circulation north of the Hadley cell. As we outlined before, the major dynamical balance will be between eddy momentum flux divergence and the Coriolis force (although surface drag is also important). Figure 9 illustrates that northward of the core of the jet stream ($\sim 30\text{N}$) in the tropopause $\frac{\partial^2 \overline{u'v'}}{\partial y \partial z} < 0$ (i.e. the meridional rate of decrease of $\overline{u'v'}$ increases with height north of the core of the jet stream below 200 hPa). From our stream function equation above we can see how this will drive a negative overturning cell. This configuration of momentum flux aloft is balanced by the Coriolis force alone, and at the surface is balanced by a combination of the Coriolis force and drag (see previous equations for QG motion on a β plane).

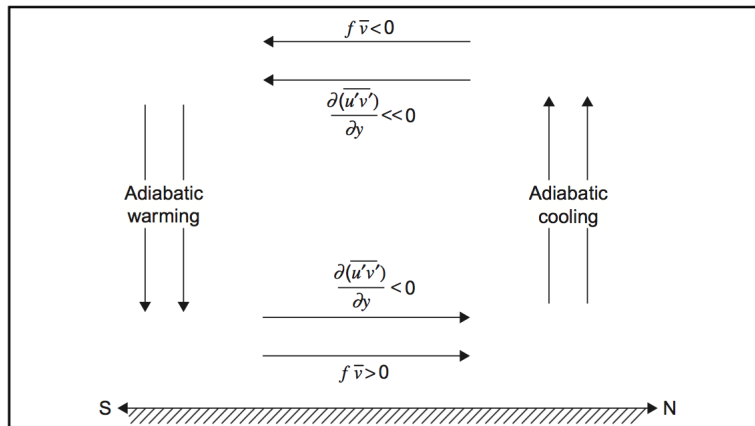


FIGURE 10.5 Schematic Eulerian mean meridional circulation forced by a vertical gradient in eddy momentum flux convergence.

Figure 10: Holton and Hakim (2012).

References

- James R Holton and Gregory J Hakim. *An introduction to dynamic meteorology*, volume 88. Academic press, 2012.
- Geoffrey K Vallis. *Atmospheric and oceanic fluid dynamics: fundamentals and large-scale circulation*. Cambridge University Press, 2006.